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Mixed strategies, uncountable times, and Pascal's Wager: a reply to Robertson

KENNY EASWARAN AND BRADLEY MONTON

1. The proponent of Pascal's Wager argues that one has pragmatic reason to believe in God, since that course of action has infinite expected utility. The mixed strategy objection holds that one could just as well follow the course of action of rolling a *n*-sided die, for arbitrarily large (but finite) *n*, and only believing in God if side #1 comes up – that course of action also has infinite expected utility. Bradley Monton (2011) has argued that mixed strategies can't evade Pascal's Wager: if one decides to follow the mixed strategy course of action, and one rolls the die and side #1 does not come up, one no longer has infinite expected utility from following that course of action, so it is rational for one to follow that mixed strategy course of action again. One can see where this process will end up, if one keeps engaging in it, so it's pragmatically rational to not keep sitting there and rolling the die, but instead to embrace the result that one fully expects to get, and choose to believe in God.

Steven Robertson (2012) has replied that some mixed strategies that meet Monton's requirements for rational agents 'do not lead inevitably to the agent believing in God, and thus avoid Monton's result.' We will explain that Robertson misses a crucial aspect of Monton's argument, and hence his argument does nothing to show that some mixed strategies can evade Pascal's Wager. We will also explain how this exchange sheds some light on the role of mixed strategies in decision theory.

2. Robertson gives three mildly complicated mixed strategies to illustrate his point, but one simple mixed strategy will do. Consider the mixed strategy of

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rolling a die, and only believing in God if #1 comes up, and if #1 does not come up one follows that strategy again. But one doubles the number of sides each time, starting with a 4-sided die, then an 8-sided die, then a 16-sided die and so on. The probability that one will believe in God is at most $1/4 + 1/8 + 1/16 + \ldots$, which equals 1/2.

The problem with that mixed strategy (and all the ones Robertson presents) is that it assumes that there will be at most a countably infinite number of die rolls. But given the standard ontology of time, there are uncountably many times – the set of times can be put in one-to-one correspondence with the set of real numbers. Monton points this out, in his original paper:

> Since [the number of sides of the die] has to be finite, you should expect with probability 1 that the side #1 will be rolled after a finite number of attempts. This finite number of attempts can take place arbitrarily quickly, since in any arbitrarily small interval of time there are an infinite number of instants – continuum-many, in fact. (Monton 2011: 643)

Given uncountably many times, the sequence of die rolls in the mixed strategies Robertson proposes will be over in an arbitrarily small amount of time. What should one do, after following the mixed strategy to completion? By Monton's argument, one should roll a die again.

This leads to the question: given uncountably many times, is there a mixed strategy that would yield the result Robertson wants, where one has infinite expected utility by following that strategy yet will not end up believing in God with probability 1? We will now prove that there is no such mixed strategy.

By the original argument that Monton gives, which Robertson grants, if there is some number n such that infinitely many of the rolls involve a die with n sides, then with probability 1, at least one of those rolls will come up with #1, leading one to believe in God. Thus, if there is a mixed strategy that *doesn't* end up giving probability 1 to believing in God eventually, it must be one such that each number n is the size of only finitely many of the die rolls. But since there are only countably many natural numbers n, this can only account for countably many of the die rolls. If there are uncountably many die rolls, then there must be some n that occurs infinitely many times, and thus if there are uncountably many die rolls, then with probability 1, one will end up believing in God, QED.

The result generalizes. If there are uncountably many times at which one pursues a mixed strategy with some non-zero chance that one will end up believing in God, then with probability 1, at least one of these plays will end with one believing in God. (We can assume that each play will have a non-zero chance of ending with belief in God, because these are the only strategies that give infinite expected utility.) This general fact holds even if the mixed strategies are generated in some way quite different from rolling die. If there are uncountably many of them, then there must be some n such that infinitely many of them give probability at least $1/n$ of believing in God, and in an infinite sequence of trials, each with probability at least $1/n$, the probability that none of them comes up positive is 0.

3. This concludes our reply to Robertson's argument. But it's worth discussing how robust Monton's argument is. What happens if we drop the assumption that there are uncountably many times? What happens if we drop the assumption that die rolls can take place arbitrarily quickly?

Let's consider what happens if we drop the assumption that there are uncountably many times. Given this restriction, Monton's argument doesn't succeed. If there are only countably many times, then there is some enumeration of them (even if the enumeration doesn't correspond to temporal sequence, which it wouldn't if the structure of time is countably dense, like the rationals). If we fix an enumeration, and roll a $2^{(n+1)}$-sided die at the nth time in the enumeration, then the probability of at least one die coming up #1 is at most 1/2. By suitably increasing the size of the die, this probability can be made as low as one wants. Thus, if there are only countably many times, Monton's original argument doesn't defeat the mixed strategy objection to Pascal's Wager.

Suppose that a die roll is such that it takes a fixed finite amount of time before one gets a result, and suppose that one is only in possession of one die at a time. This is an instance of what Monton called a 'delayed strategy' in his original article (2011: 645). If it takes 50 years to get a die roll result, then it is no surprise that one can choose a sequence of die rolls such that one might never end up believing in God. Delaying 1 second between results, or 1 nanosecond, is just a difference in degree. But a referee for this journal has pointed out that there may be a difference in kind here. When delaying 50 years between rolls, one is choosing at intermediate times not to even play a mixed strategy involving belief in God, but instead one is just waiting. However, if each die roll takes a full second, then there may be only countably many total rolls, even though one is pursuing the mixed strategy at every point in time.

This objection takes too seriously the physical limitations on the agent. Just as the agent won't actually live forever, the agent actually has to take some finite amount of time for each die roll, and the agent actually has some finite upper bound on the size of die she can roll. The force of our argument is that the agent recognizes that *if* she lived longer, then she ought to roll more times, and *if* she could roll faster, then she ought to roll more times, and so she should believe in the way that her idealized self would after taking a limiting idealization. Thus, *even if* she could roll an arbitrarily large die, the idealization Robertson suggests, Monton's previous idealizations would still leave her with probability 1 of eventually believing, so she might as well believe already, even though she is not ideal.

In sum: Monton made these two assumptions explicit in his original argument, that the die rolls can take place arbitrarily quickly, and that there are uncountably many times. These two assumptions are necessary for his argument to succeed, and they are sufficient to respond to Robertson's objection.

4. We close with one final point. This entire exchange shows a weakness of applying the notion of 'mixed strategies' outside the original game-theoretic context for which it was designed. Von Neumann (1928), in the article that invented the field of game theory and introduced the concept of a 'mixed strategy', showed that in a two-player, zero-sum game of perfect information, each player has a mixed strategy that minimizes her maximum possible loss. Nash (1950) showed further that in a two-player, non-zero-sum game of perfect information, there is always a Nash equilibrium, if one allows players to choose mixed strategies. However, it is important in this game-theoretic context that each player has just one chance to play. A mixed strategy therefore must involve randomizing at the moment of play – if the player randomizes at some earlier point, then in-between the moment of randomization and the moment of play, the player will not see herself as playing a mixed strategy, and will therefore be violating the rule of playing a minimax strategy or Nash equilibrium strategy. (The normative force of these rules is far beyond the scope of this article.) Thus, since there is only one time at which each player randomizes, the types of scenarios considered above don't arise. The player may do any random actions she likes at all the uncountably many times that don't involve actually playing the game, but all of this will be overridden by the final randomization at the moment of play.

For a situation that doesn't involve a single moment of play, however, this doesn't apply. The decision to believe in God (to the extent that it is really a decision) is a decision that has no single moment of play. Monton's original article suggests (as does Pascal's) that the decision is a sort of one-way gate – once one decides to believe, there is no going back, but for unbelievers, there are always future chances to believe (at least, until death). Given the potential for such an irreversible decision, any permissible (non-delay-based) mixed strategy will guarantee that one will end up in the state that never reverses – in this case, the state of believing in God. Note that, if neither state is permanent, then the parameters of the debate over Pascal's Wager are completely different – neither believing nor disbelieving at a point in time involves one in any serious commitment.

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A new argument for animalism

STEPHAN BLATTI

Though Aristotelian in spirit, the view known as *animalism* is a relative latecomer to the debate over personal identity, having been defended only within the past 25 years or so.[1] Its advocates make the following straightforward claim: we are animals. According to the intended reading of this claim, the 'are' reflects the 'is' of numerical identity (not the 'is' of non-identical constitution); the 'we' is intended to pick out you, me and others of our kind; and 'human animals' is meant to refer to biological organisms of the *Homo sapiens* species. According to animalism's most sophisticated rival, *neo-Lockean constitutionalism*, we persons are non-identically constituted by human animals, rather like the way statues are said to be constituted by the lumps of matter with which they coincide.[2]

The standard argument for animalism is commonly known as the *Thinking Animal Argument* (TAA). Very roughly, TAA registers the implausible multiplication of thinkers to which anyone who rejects animalism is committed.[3] Recently, however, a structurally analogous line of argument has been shown

1 The founding advocates of this view include Ayers (1991), Carter (1989), Olson (1997), Snowdon (1990) and van Inwagen (1990). Other notable proponents include DeGrazia (2005), Hershenov (2005), Mackie (1999), Merricks (2001) and Wiggins (2001).

2 The leading defenders of neo-Lockean constitutionalism are Baker (2000), Johnston (1987) and Shoemaker (1999, 2011). Other opponents of animalism – neo-Lockeans who regard psychological properties as essential, but who are not constitutionalists – include Hudson (2001), Lowe (1996), McMahan (2002), Noonan (1998) and Parfit (2012). A more detailed overview of the animalism debate can be found in Blatti (forthcoming).

3 I believe that Snowdon (1990: 91) was the first to advance this argument at a 1986 conference. Later presentations can be found in Ayers 1991, vol. 2: 283, Carter 1988, Olson 1997: 106–9 and elsewhere. What is now the standard exposition of this argument can be found in Olson 2003.

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